How Much Can I Make? Insights on Belief Updating in the Labor Market

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BYU Economics Graduate Student Conference
6 June 2024

Introduction

Introduction

- Canonical job search models typically assume the agent knows the wage distribution (McCall 1970, Mortensen 1970, Weitzman 1979).
- Search models with unknown underlying distribution (Rothschild 1978, Rosenfield & Shapiro 1981, Talmain 1992, Li & Yu 2018). $\Rightarrow$ Action is a function of beliefs
- The empirical literature has primarily focused on the first class of models due to the absence of good-quality data on beliefs. $\Rightarrow$ More research needed

Research Question

How do people update their beliefs about the wage offers they will receive in the future?

Bayes' Rule

$$g^{bayes}_{t+1}(\theta | x_t) = \frac{\overbrace{g_t(\theta)}^{\text{prior}}\ \overbrace{p(x_t | \theta)}^{\text{signal}}}{\underbrace{\int_{\theta' \in \Theta} g_t(\theta') p(x_t | \theta')}_{\text{normalizing factor}}}$$

Review paper on lab experiments by Benjamin (2019) shows robust evidence of **non-Bayesian updating**

Related Literature

1. **Non-Bayesian Updating:** (Grether 1980, Epstein, Noor & Sandroni 2010, Hagmann & Loewenstein 2017, Benjamin, Bodoh-Creed & Rabin 2019)
2. **Field Evidence of Non-Bayesian Updating:** (DellaVigna 2009, Conlon, Pilossoph, Wiswall & Zafar 2018, Bordalo, Gennaioli, Porta & Shleifer 2019, Augenblick & Rabin 2021)
3. **Job Search with Learning:** (Kudlyak, Lkhagvasuren & Sysuyev 2014, Conlon, Pilossoph, Wiswall & Zafar 2018, Mueller, Spinnewijn & Topa 2021, Potter 2021, Jiang & Zen 2023) $\Longrightarrow$ Most assume Bayesian updating

Contributions Relative to Most Similar Paper

- Conlon, Pilossoph, Wiswall & Zafar (2018) also uses the SCE (same dataset) and find that people are over-updating relative to the Bayesian benchmark (similar results).
- **Overview:** Conlon et al. (2018) is interested in how information frictions affect job search, we are interested in how people update their beliefs.
- **Methodological:** Our approach makes more conservative assumptions and we approach the problem from a theoretical angle. (more later)
- Our additional analyses on policy relevance also reveal a new pattern in the data.

Data

Background: SCE Survey

- Survey conducted by NY Fed
- A representative American population
  $\Longrightarrow$ Most people are employed and not searching
- Each person can be surveyed up to 3 times, with a 4-month gap between each survey

Data Overview

Statistic	Number of Individuals	Average Number of Surveys per Individual
All Individuals	2,596	2.4141
Ever Got Offer(s)	576	2.4809
Never Got Offers	2,020	2.3951
Ever Searched	724	2.4986
Never Searched	1,680	2.3774
Missing Search Info	192	2.4167
Data Range	3/2015-11/2019	

Table: Dataset composition. Observations from one of the surveys in 2020 were excluded to avoid measuring the effects of the pandemic on expectations. Measures refer to offers received/searching done between consecutive surveys.

Best offer estimate: Question text

OO2a2 - OO2a2 (Added March 2015)

Think about the job offers that you may receive within the coming four months. Roughly speaking, what do you think the annual salary for the best offer will be for the first year?

Note the best offer is the offer you would be most likely to accept.

________________________ dollars

Best offer distribution: Question text

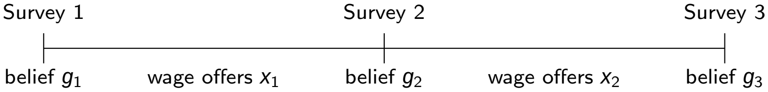
OO2b - OO2b (shown if OO2a2 > 0 each response is % of OO2a2 ranging from .8 to 1.2) (Added November 2014)

Think again about the job offers that you may receive within the coming four months. What do you think is the percent chance that the job with the best offer will have an annual salary for the first year of...

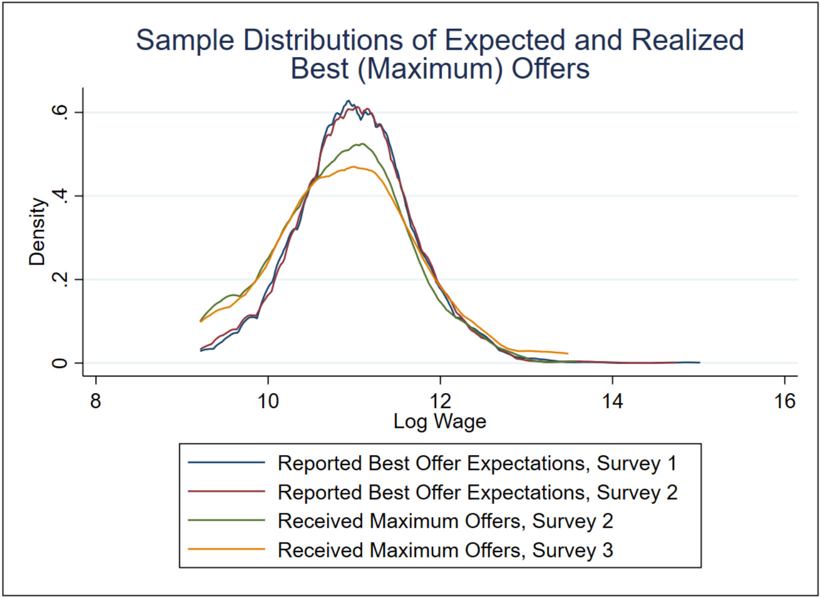
The best offer is the offer you would be most likely to accept.

Less than [0.8* OO2a2] dollars (1)	________________	% (1)
Between [0.8* OO2a2] dollars and [0.9* OO2a2] dollars (2)	________________	% (2)
Between [0.9* OO2a2] dollars and [1.0* OO2a2] dollars (3)	________________	% (3)
Between [1.0* OO2a2] dollars and [1.1* OO2a2] dollars (4)	________________	% (4)
Between [1.1* OO2a2] dollars and [1.2* OO2a2] dollars (5)	________________	% (5)
More than [1.2* OO2a2] dollars (6)	________________	% (6)

Survey Timeline



Data Quality



Data Structure: Simplified Example

- **Period 1:** Guess of average best wage offer is $100

	$90 < w$ 100	$100 < w$ 110	$110 < w$ 120
$p(\ )$	0.4	0.2	0.4

Table: Period 1 Beliefs

- **Period 2:** Guess of average best wage offer is $90

	$81 < w$ 90	$90 < w$ 99	$99 < w$ 108
$p(\ )$	0.3	0.4	0.3

Table: Period 2 Beliefs

Data Structure: Simplified Example

- **Period 1:** Guess of average best wage offer is \$100

	w 90	w 99	w 100	w 108	w 110	w 120
$p(\ )$			0.4		0.6	1

Table: Period 1 Beliefs

- **Period 2:** Guess of average best wage offer is \$90

	w 90	w 99	w 100	w 108	w 110	w 120
$p(\ )$	0.3	0.7		1		

Table: Period 2 Beliefs

Data Structure: Simplified Example

- **Period 1:** Guess of average best wage offer is \$100

	w_{90}	w_{99}	w_{100}	w_{108}	w_{110}	w_{120}
$p(\ )$			0.4		0.6	1

Table: Period 1 Beliefs

- **Period 2:** Guess of average best wage offer is \$90

	w_{90}	w_{99}	w_{100}	w_{108}	w_{110}	w_{120}
$p(\ )$	0.3	0.7	0.9	1	1	1

Table: Period 2 Beliefs

Distribution Fitting

- We fit the distribution using Simulated Method of Moments.
  1. Log-normal to max wage distribution
  2. Gumbel distribution to max wage distribution
  3. Log-normal to individual wage distribution (recovered using another question about expected number of offers)
  4. Kernel Density Estimation
- **Selection Criteria:** Smallest mean-squared error in fitting.

Theory

Martingale Property

- Let the state space be Θ and set of signals X
- **Martingale Property:** $\mathrm{E}_X(g_{t+1}(\ |x)|g_t(\ )) = g_t(\ )$
- Idea: before you observe the signal, you should not expect your beliefs to change

Competing Behavioral Models of Updating

1. Bayesian updating

$$g_{t+1}^{bayes}(\ j x_{t+1}) = \frac{g_t(\ )p(x_{t+1} j\ )}{R_{\ ^0 2 \Theta}\, g_t(\ ^0)p(x_{t+1} j\ ^0)}$$

2. Affine Transformation of Prior and Bayesian Belief (Epstein, Noor & Sandroni 2010)

$$g_{t+1}^{bias}(\ j x_{t+1}) = (1 \quad\ )g_t(\ ) + \ g_{t+1}^{bayes}(\ j x)$$

3. Exponential Non-Bayesian updating Grether (1980)

$$g_{t+1}^{bias}(\ j x_{t+1}) = \frac{g_t(\ )^a p(x_{t+1} j\ )^b}{R_{\ ^0 2 \Theta}\, g_t(\ ^0)^a p(x_{t+1} j\ ^0)^b}$$

4. Convex Combination of Reference Belief and Bayesian Belief (Hagmann & Loewenstein 2017)

$$g_{t+1}^{bias}(\ j x_{t+1}) = (1 \quad\ )\ (\ ) + \ g_{t+1}^{bayes}(\ j x)$$

Simple Model

Red Die

Value of die roll	Tokens awarded
1	10
2	20
3	20
4	30
5	30
6	30

Blue Die

Value of die roll	Tokens awarded
1	10
2	10
3	10
4	20
5	20
6	30

- One of the dice is selected randomly.
- You can only observe the tokens awarded, not the color of the die.
- Given a 50-50 prior, the expected value from rolling a die is 20.
- How will your expectation change after observing a 10-token outcome?

Uncertainty in the Environment

- The agent believes there is a set of possible wage distributions, F (color of die)
- True wage distribution is in F which are indexed by an ordered set $\Theta \quad \mathbb{R}$
- The agent has a full-support belief g_t over at time t
- We partition the wages into n wage bins $\{[a_0, a_1), [a_1, a_2), \ldots [a_{n-1}, a_n)\}$. At bin i,

$$ {}^{i}_{t} = \overbrace{\int_{{}' \in \Theta} g_t({}') \underbrace{\int_{a_{i-1}}^{a_i} f(w|{}')dw}_{\text{Probability of drawing a wage within the bin from distribution }{}'} d{}'}^{\text{Averaged over beliefs of distributions}} $$

- **Remark:** If the updating rule used to update g has the Martingale property, then so will .

Empirical Strategy

2 States (Augenblick and Rabin, 2021)

- Overview: Model of belief dynamics (multiple time periods)
- 2 states and state 1 occurs with probability
- Belief movement

$$m_{t_1;t_2} \quad \sum_{=t_1}^{t_2 \; 1} (\quad {}_{+1} \quad)^2$$

- Uncertainty Reduction

$$r_{t_1;t_2} \quad \sum_{=t_1}^{t_2 \; 1} \quad (1 \quad) \quad {}_{+1}(1 \quad {}_{+1})$$

$$= \quad {}_{t_1}(1 \quad {}_{t_1}) \quad {}_{t_2}(1 \quad {}_{t_2})$$

Many States (Augenblick and Rabin, 2021)

- n states and state i occurs with probability ^{i}
- Belief movement

$$m_{t_1;t_2} \quad \sum_{i=1}^{n} \sum_{=t_1}^{t_2 \; 1} (\; ^{i}_{+1} \quad ^{i} \;)^2$$

- Uncertainty Reduction

$$r_{t_1;t_2} \quad \sum_{i=1}^{n} \sum_{=t_1}^{t_2 \; 1} \; ^{i} (1 \quad ^{i}) \quad ^{i}_{+1} (1 \quad ^{i}_{+1})$$

$$= \sum_{i=1}^{n} \; ^{i}_{t_1} (1 \quad ^{i}_{t_1}) \quad ^{i}_{t_2} (1 \quad ^{i}_{t_2})$$

Test For Bayesianism (Martingale)

We denote $M_{t_1;t_2}$ and $R_{t_1;t_2}$ as the random variable for belief movement and uncertainty reduction respectively

Statistical test: $EM_{t_1;t_2} = ER_{t_1;t_2}$

Intuition: When there is a small uncertainty reduction (not many or weak signals or a tight prior), belief movement should be small.

$$X = \overline{m}_{1;2} - \overline{r}_{1;2}$$

$$Z \quad \frac{\sqrt{n}}{s_{1;2}}(\overline{m}_{1;2} - \overline{r}_{1;2}) \xrightarrow{n \to \infty} N(0;1)$$

$$X_{norm} = \frac{\overline{m}_{1;2}}{\overline{r}_{1;2}}$$

Di erences with Conlon et al. (2018)

Methodology: Conlon et al. (2018) assumes a Gaussian updating framework for their non-Bayesian analysis. They assume:

- Priors are log-normally distributed
- Signals (wage o ers) are log-normally distributed
- People correctly perceived the variance of the signal
- Stable wage distribution during survey period

These assumptions allows them to compute a Bayesian benchmark (Result: Overupdating relative to Bayesian Benchmark)

Di erences with Conlon et al. (2018)

Our method uses fewer assumptions. We only assume:

- Priors are log-normally distributed (for distribution tting)
- ~~Signals (wage offers) are log-normally distributed~~
- ~~People correctly perceived the variance of the signal~~
- Stable Wage Distribution during survey period

These give us the following bene ts:

- Our analysis does not need a Bayesian benchmark and we require fewer assumptions
- We can use more observations where there is no job o er
- We can reject all updating rule with martingale property (more than just Bayesian)

Assumptions of the Test

Assumption on the distribution of beliefs (tried other distribution)

Stable wage distribution: No exogenous shock to the wage distribution during the survey period

Wage Distribution Stability Tests

- We run a xed e ects regression to see whether individuals' o ers systematically di er across survey period (necessary but not su cient).
- We also pool all the received o ers together from the second and third surveys and nd the Kullback{Leibler divergence.

Wage stability tests

Wage Distribution Stability Tests: Takeaways

- Not many observations with o ers in consecutive periods.
- Possibly signi cant increases in wage distribution on third survey, but very few observations to make this comparison.
- Main results very similar when dropping individuals with three surveys.
- Four-month period between surveys should be a relatively short time for individual wage distributions to change for many of the individuals in the survey.

Martingale Test Results

Martingale Test

Test Results

Statistic	All Individuals	Got O er?		Searched?	
		Yes	No	Yes	No
$\overline{m}$	.9341	1.0499	.9011	1.0083	.9004
	(.0193)	(.0434)	(.0213)	(.0372)	(.0213)
$\overline{r}$	.1805	.1981	.1755	.1989	.1687
	(.0079)	(.0190)	(.0085)	(.0134)	(.0080)
X	.7536	.8518	.7256	.8094	.7317
	(.0207)	(.0472)	(.0248)	(.0358)	(.0224)
X_{norm}	5.1751	5.2998	5.1345	5.0694	5.3373
Observations	2489	552	1937	691	1613

Table: Excess movement statistics: Log normal- tted results. Standard errors in parentheses.

Robustness Checks

- We do two analyses to check the robustness of our results
  - Check whether results are due to measurement error
    Measurement error robustness
  - Check whether results are influenced by bin definitions
    Binary bin robustness
- We find that the test results are robust to both

Competing Behavioral Models of Updating

We can reject updating rules with Martingale property

Bayesian updating

$$g_{t+1}(\ \mid x_{t+1}) = \frac{g_t(\ )p(x_{t+1} \mid \ )}{R_{\ ^0 2}\ g_t(\ ^0)p(x_{t+1} \mid \ ^0)}$$

A ne Transformation of Prior and Bayesian Belief (Epstein, Noor & Sandroni 2010)

$$g^{bias}_{t+1}(\ \mid x_{t+1}) = (1 \qquad)g_t(\ ) + \quad g^{bayes}_{t+1}(\ \mid x)$$

Updating Rules (Cont.)

Exponential Non-Bayesian updating Grether (1980)

$$g^{bias}_{t+1}(\ \ j x_{t+1}) = \frac{g_t(\ \ )^a p(x_{t+1}\, j\ \ )^b}{R_{{}^0 2}\ g_t(\ {}^0)^a p(x_{t+1}\, j\ {}^0)^b}$$

Convex Combination of Reference Belief and Bayesian Belief (Hagmann & Loewenstein 2017)

$$g^{bias}_{t+1}(\ \ j x_{t+1}) = (1\qquad)\ \ (\ \ ) + \quad g^{bayes}_{t+1}(\ \ j x)$$

Result Interpretation

Excess belief movement is consistent with overreacting to signals and base-rate neglect in the Grether (1980) model.

$$g_{t+1}^{bias}(\theta | x_{t+1}) = \frac{g_t(\theta)^a p(x_{t+1} | \theta)^b}{\int_{\theta' \in \Theta} g_t(\theta')^a p(x_{t+1} | \theta')^b}$$

- Base-rate neglect: $0 \leq a < 1$ and/or overreaction: $b > 1$
- Hagmann & Loewenstein (2017) can also produce excess belief movement
- Due to the excess belief movement, we postulate that information provision policy is going to be potent

Do we need policy intervention?

Base-rate neglect or overreaction

- Ideally, we would observe individuals over more time periods and check whether beliefs converge
  - Decrease in belief movement and/or uncertainty reduction over time
- We have at most two updates for any individuals, this can only give us suggestive evidence.
- Our analysis in the next slide does not nd signi cant changes in either between periods (but note that these are not precise zeros).

Fixed E ects Regression

	Movement	Reduction
Second Update	0.0121	-0.0131
	(0.0196)	(0.0156)
Constant	0.624***	0.171***
	(0.00979)	(0.00780)
Observations	2,146	2,146
R-squared	0.000	0.001
Number of userid	1,073	1,073

Robust standard errors in parentheses
*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Results from Lab Experiments

A recent survey paper Benjamin (2019) found mostly conservatism and base-rate neglect in lab experiments.

Asymmetric Updating

Another factor that determines the need for policy intervention is asymmetric updating.

If individuals only selectively update depending on the type signals they receive, their beliefs might not converge to the true distribution.

Asymmetric updating is the main feature of the Hagmann & Loewenstein (2017) model

Survey Question

Direction and Normalization Definitions

Define $\bar{x}_t$ as the average wage offer reported in period t's survey and $\hat{x}_t$ as the expected average wage elicited period t survey.

Normalized Difference in Expectations:

$$\frac{\hat{x}_2 - \hat{x}_1}{|\bar{x}_2 - \hat{x}_1|}$$

Signal Direction:

- Positive Signal: $\bar{x}_2 > \hat{x}_1$
- Negative Signal: $\bar{x}_2 < \hat{x}_1$

Normalized Di erences

Normalized Di erences

Discussion

- Papers have found that people do not adjust their reservation wage downwards fast enough (Krueger & Mueller 2016)
- We have a similar result where beliefs are not as responsive when a negative signal is realized, movement in beliefs primarily comes from positive signals

Conclusion

Conclusion

- We perform an excess belief movement (Augenblick & Rabin 2021) and reject that people are updating their beliefs with rules that satisfy the martingale property
- We found large excess movement (information provision is likely to move beliefs, at least in short-run)
- No evidence for converging beliefs (base-rate neglect possibly present), but panels short and results not statistically signi cant
- Evidence of asymmetric updating where people update more when they receive a positive signal

FE Regression: All Wage O ers

Number of Surveys with Wage O ers	2	3
2nd Survey with Wages	-429.7	1,569
	(2,393)	(2,702)
3rd Survey with Wages		3,606*
		(1,868)
Constant	70,102***	82,902***
	(1,200)	(1,272)
Observations	696	284
R-squared	0.000	0.009
Number of userid	213	49

Robust standard errors in parentheses
*** p < 0.01, ** p< 0.05, * p< 0.1

Table: Individual-level xed e ect regression of o er wage on dummies for number of surveys taken in current period. Results estimated separately for individuals who report o ers on only two or on three consecutive surveys. Standard errors are clustered at the state level.

Period Wage Distributions: Graph

Period Wage Distributions: Pooled Graph

Period Wage Distributions: K-L Divergence

Statistic	Value
Entropy	0.0378
95% Con dence Interval	[0.0075, 0.0680]
Histogram Bins	20
Comparison Observations	532
Reference Observations	448

Table: Kullback{Leibler Divergence Statistics for Period Log Wage O er Distributions.

Back

Measurement Errors

Measurement Error can potentially explain excess belief movement even when the agent is Bayesian Augenblick & Rabin (2021)

Consider that the agent has a true belief of ${}_t^i$ but reports a distorted belief $\hat{}_t^i = {}_t^i + {}_t^i$, where ${}_t^i$ is the measurement error.

Assume that the measurement error term is mean zero with variance ${}^{i\,2}$ and uncorrelated with belief and error realizations ($E({}_t^i\,{}_t^i) = E({}_t^i\,{}_{t-1}^i) = E({}_t^i\,{}_{t-1}^i) = 0$)

We show in our appendix that expected excess belief movement will be equal to $X = \sum_{i=1}^{n} 2\,{}_t^{i\,2} \neq 0$

Measurement Errors

- Monte Carlo Simulation: 10000 simulations
- Same number of observations as in our dataset.
- 6 states.
- Prior is uniform distribution.
- We pick a posterior distribution that is Bayes' plausible to match the uncertainty reduction.
- Measurement error is simulated by drawing from a Dirichlet distribution with mean centered around the correct beliefs.
- $\Delta \quad \sum_{i=1}^{6} |\hat{}^{i}_{1} \quad {}^{i}_{1}|$

Measurement Errors

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- We pick a posterior distribution that is Bayes' plausible to match the uncertainty reduction.
- Measurement error is simulated by drawing from a Dirichlet distribution with mean centered around the correct beliefs.
- $\Delta \quad \sum_{i=1}^{6} j\hat{\ }^{i}_{1} \quad {}^{i}_{1}j$
- $0 \quad \Delta \quad 2$

Measurement Errors

Simulated Statistic	Uniform Prior with Matched Parameters	Target Value from Data
$\overline{m}$	0.9364 [0.9183, 0.9540]	0.9364
$\overline{r}$	0.1805 [-0.2379, 0.4546]	0.1825
X	0.7539 [0.7312, 0.7758]	0.7540
X_{norm}	5.1341 [4.8726, 5.4023]	5.1310
Δ	1.162 [1.1520, 1.1714]	

$\Rightarrow$ Two-bin example: True belief is 70 for bin 1 but 12 was reported. Measure error unlikely to the only explanation! Back

Other Studies

- The excess belief movement test has been applied to forecasting geopolitical events, sports betting (Augenblick & Rabin 2021) and financial markets (Augenblick, Lazarus & Thaler 2023)
- The largest statistic was a normalized excess movement of 1.46
- Most of these studies have a binary state

Test Results with Two Wage Bins

- Combine the bins below the first response and the bins above it

Statistic	Two Bins	Six Bins
$\overline{m}$	0.6058	0.9341
	(0.0172)	(.0193)
$\overline{r}$	0.1072	0.1805
	(0.0064)	(.0079)
X	0.4986	0.7536
	(0.0149)	(.0207)
X_{norm}	5.6511	5.1751

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